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## Capitalization and Community Income Distributions


Charles A. M. de Bartolome

# CAPITALIZATION AND COMMUNITY INCOME DISTRIBUTIONS

by

Charles A.M. de Bartolome
*University of Colorado*

and

Stephen L. Ross
*University of Connecticut*

ABSTRACT

Tiebout's (1956) model of fiscal competition suggests income sorting between communities while the Alonso (1964), Mills (1967) and Muth (1969) model of the monocentric city suggests income sorting over space. We add fiscal decentralization to the spatial model by considering a circular inner city surrounded by a suburban community. The fiscal difference between the communities and the commuting advantage of locations closer to the city center are capitalized into house prices. The model has equilibria in which there is income sorting between communities and equilibria in which there is income mixing between communities. The structure allows for the possibility of undeveloped land in the inner city.

JEL Classification: H73, R12, R14.

Key words: Communities, income heterogeneity, undeveloped land, capitalization.

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Address for correspondence: Charles de Bartolome, Department of Economics, Campus Box 256, University of Colorado, Boulder CO 80309-0256. e-mail: debartol@spot.colorado.edu

# INTRODUCTION

An important issue in local public economics is whether a household's mobility within a metropolitan area leads to communities in which residents have similar incomes. In Tiebout's (1956) model of fiscal competition, communities are formed on a featureless plain and community boundaries may be freely adjusted. Each community provides a public service which is financed by a head-tax. A household's income does not depend on the community in which it resides. A household shops over communities, choosing the community which provides his preferred public service level. If the public service is a normal good, households with different incomes demand different public service levels. In consequence, households with different incomes chose different communities, or all households within each community have the same income (McGuire (1974), Berglas (1976a) and Wooders (1978)). The prediction of households sorting themselves by income between communities is income elasticities of land demand and of commuting cost (Wheaton (1977)). If land demand is unresponsive to income changes and commuting costs increase with income, rich households outbid poor households for locations closer to the city's center. Conversely, if land demand is sufficiently income elastic, the saving achieved by the purchase of land further from the city's center is greater for the rich households and compensates them for the associated increase in commuting cost. In this case rich households choose to live in the low-priced locations further from the city's center. In both cases the prediction is of a monotonic relationship between household income and distance from the metropolitan center.[2] Therefore, if the metropolitan area is considered to be a system of annular communities, the model's prediction for income distribution is similar to that of the Tiebout model - incomes in each community lie in an interval and the income intervals associated with different communities do not overlap.

The model of income sorting between communities underlies much public policy. For example, programs which redistribute resources from rich school districts to poor school districts are often justified as income redistribution programs, it being considered self-evident that only rich households live in the rich school districts and only poor households live in the poor school districts. However, the prediction of strict income sorting does not fit well with the data. A significant percentage of families in both the inner city and in the suburbs have income below the poverty level (14.1% and 6% respectively in 1989).[3] Pack and Pack (1977 and 1978) find larger income variation within the towns of the metropolitan areas of Pennsylvania than is consistent with the homogeneous communities predicted by the Tiebout model. Persky (1992) examines Chicago and finds considerable evidence of income heterogeneity in both the city and the suburbs. Epple and Platt (1998) discuss the income variation within the Boston metropolitan area: the incomes of the wealthiest households in a community of low average income typically exceed the incomes of the poorest households in a community of high average income.

In view of the large amount of federal and state aid directed at inner cities and of observed income heterogeneity within communities, a model which can predict income mixing between communities seems desirable. Various modifications have been made to Tiebout's model to ensure income heterogeneity within a community. Berglas (1976b), Stiglitz (1983), McGuire (1991) and Brueckner (1994) consider households to earn their incomes at firms which are located in the community. The firms have a production technology which requires the use of low- and high-skilled workers. In such a situation, which is perhaps best exemplified by the nineteenth century company town, low-wage and high-wage households coexist in the same town. Berglas and Pines (1981) suggest that communities provide several different public services and that the optimal community size for each public service is different. With the optimal community size for one public service being less than for another public service, it is desirable to add to the community households who are relatively large users of the second public service. Epple and Platt (1998) create income mixing by allowing households to differ both in their incomes and in their preferences for the public service. A community providing an intermediate level of the public service is chosen both by rich households who place a low weight on the public service and by poor households who place a high weight on the public service.

Although the "pure" models of fiscal competition and of the monocentric city both give similar predictions of income sorting between communities, we show that a model which combines elements of both models can predict income mixing between communities. We consider a metropolitan area to be comprised of a circular inner city surrounded by the suburbs. At the center of the inner city is the central business district to which all households commute. We consider two income classes. A household's cost of commuting is proportional to the distance traveled and to its income, and land demand is wealth inelastic. These assumptions ensure that within each community rich households live closer to the metropolitan center. There is always an equilibrium with income sorting between communities: at least one community contains only one income level. This equilibrium may be associated with undeveloped land in the inner city. We also find some equilibria with income mixing between communities: both communities contain both income levels. In these equilibria, in the inner city poor households form the la4d1politanto th city pthe cenobecae city y valueity phigh ho Tc 0t ilibBi6a city ithe cenomoin eacnolds 851 T

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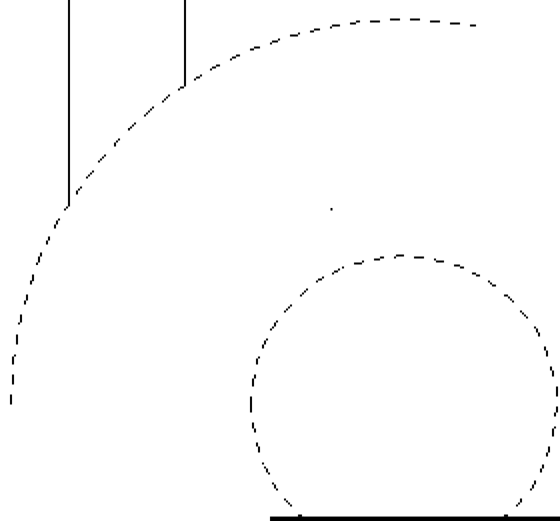
to rich households of the smaller commute. As a result, a net surplus - commuting benefits less the increase in house prices - is created for rich households as the location in the inner city moves away from the suburban boundary and towards the metropolitan center. At a certain distance from the boundary this surplus equals the compensation required by rich households for the lower public service, and rich households are indifferent between such locations and their suburban locations. Capitalization similarly allows poor households to be indifferent about living in either community.

The income distribution in the equilibrium with income mixing is: in the inner city rich households live near the center and are surrounded by a ring of poor households. In the suburbs, adjacent to the boundary between the inner city and the suburbs, is a ring of rich households and these households are in turn surrounded by a ring of poor households. As the location moves out from the metropolitan center, household income falls, then rises at the boundary between inner city and suburbs, and then falls again. This distribution is descriptively similar to the empirical relationship found for “old” cities by Glaeser at al. (2000).

The paper is organized as follows. In Section 2 lays out the model and proves the existence of an equilibrium with income sorting. The presence of undeveloped land is highlighted. Section 3 presents the equilibrium with income mixing. Section 4 presents conditions sufficient to ensure the existence of an equilibrium with income mixing. Some welfare results are discussed in Section 5: we show that poor households may achieve higher utility in the equilibrium with income mixing than in the equilibrium with income sorting. Section 6 concludes.

## 2. INCOME SORTING BETWEEN COMMUNITIES

Figure 1.M1 5*42Ln (Figure 794-2 En c0.88polian.0.3r TD - 80a) Tj ..0i2475 0489Sl 5-845



The use of a model in which a sorting equilibrium exists in general provides a simple basis for examining the sufficient conditions for the existence of an equilibrium with income mixing.

A household's housing choice within the community is restricted to its location. The household takes the housing price schedule *r(s)* and the public service *g* in a community as given. His preferred location *s* within the community solves

$$\max_{s} \; U(M^{h} - tM^{h}s - r(s) - g + T) + V(g).$$

The benefit of locating closer to the metropolitan center is greater for the rich household or, within a cost would change only marginally. This is formalized in Lemma B.[9]

LEMMA B: *If $s_A$ and $s_B$ both lie in a given community and $s_A < s_B$, then $r(s_A) > r(s_B)$ and $r(s)$ is a continuous function from $s_A$ to s*

$$\pi x_u^2 = (1 - \theta) N\alpha . \qquad (2)$$

The reservation price of land is $r_0$ $(r_0\ \$0)$ . Figure 2 shows possible rent schedules The curves may also be interpreted as the bid-rent curves of the households. Consider Figure 2(a) in which there is no undeveloped land. For poor households living at the suburban fringe, the rent is $r_0$. As the location moves inwards, the commuting advantage to the poor household is capitalized so that the rent rises at rate $tM_1$ or the rent schedule is *AB*. The public service (and associated taxes) changes discretely as the location moves across the urban boundary at distance *B* from the metropolitan center. Poor households vote their desired public service in the suburbs but in the urban area it is set by rich households. Hence, as the location moves across the urban boundary, rent falls by the amount represented by the line segment *BC* to reflect the cost to the poor household of the higher urban public service. Poor households live in the outer urban area and the rent gradient along *CD* is $tM_1$ . However, at distance $x_u$ from the metropolitan center, households become rich and the rent gradient along *DE* increases to $tM_2$ to reflect the advantage to them of a marginally smaller commute.

The reservation price $r_0$ is a rent floor. In Figures 2(b) and 2(c), the cost to the poor household of the high urban public service is sufficiently large that poor households are unwilling to pay rent $r_0$ to live at the boundary of the urban area. As the location moves inward, the benefit of the smaller commute increases and, in Figure 2(b), the location becomes attractive to poor households at distance *X* from the metropolitan center. In Figure 2(c), even at distance $x_u$ a poor household is unwilling to pay rent $r_0$ and there are no poor urban households.

Poor households live at the suburban fringe where the price of land is the reservation price $r_0$ . Therefore, for poor households living in the suburbs[10]

$$c_{1r} = r_0 + [illegible] Y.$$

If there is no undev[illegible]oped urba[illegible] (cas[illegible]e rent at the urban boundary must be at le[illegible]

$$x_u < X = [illegible] : \quad c_{1u} \geq r [illegible] B [illegible]$$

If there is undev[illegible]loped urban l[illegible]t the limit of urban development is $r_0$. If poor ho[illegible]

live at the limi[illegible] of urban develo[illegible] ([illegible] *b*)

$$x_u [illegible] \quad c_{1u} [illegible]$$ (4c) [illegible] (4d)



If there are no poor households in the urban area, it is the rich households who live at the limit of urban development (case *c*) and

If poor households live in each community, they achieve the same utility in either community, or

$$x_u < X \leq B: \qquad . \qquad \text{(6a)}$$

[redacted] the nine variables $c_{2u}, c_{1u}, c_{1s}$ [redacted]

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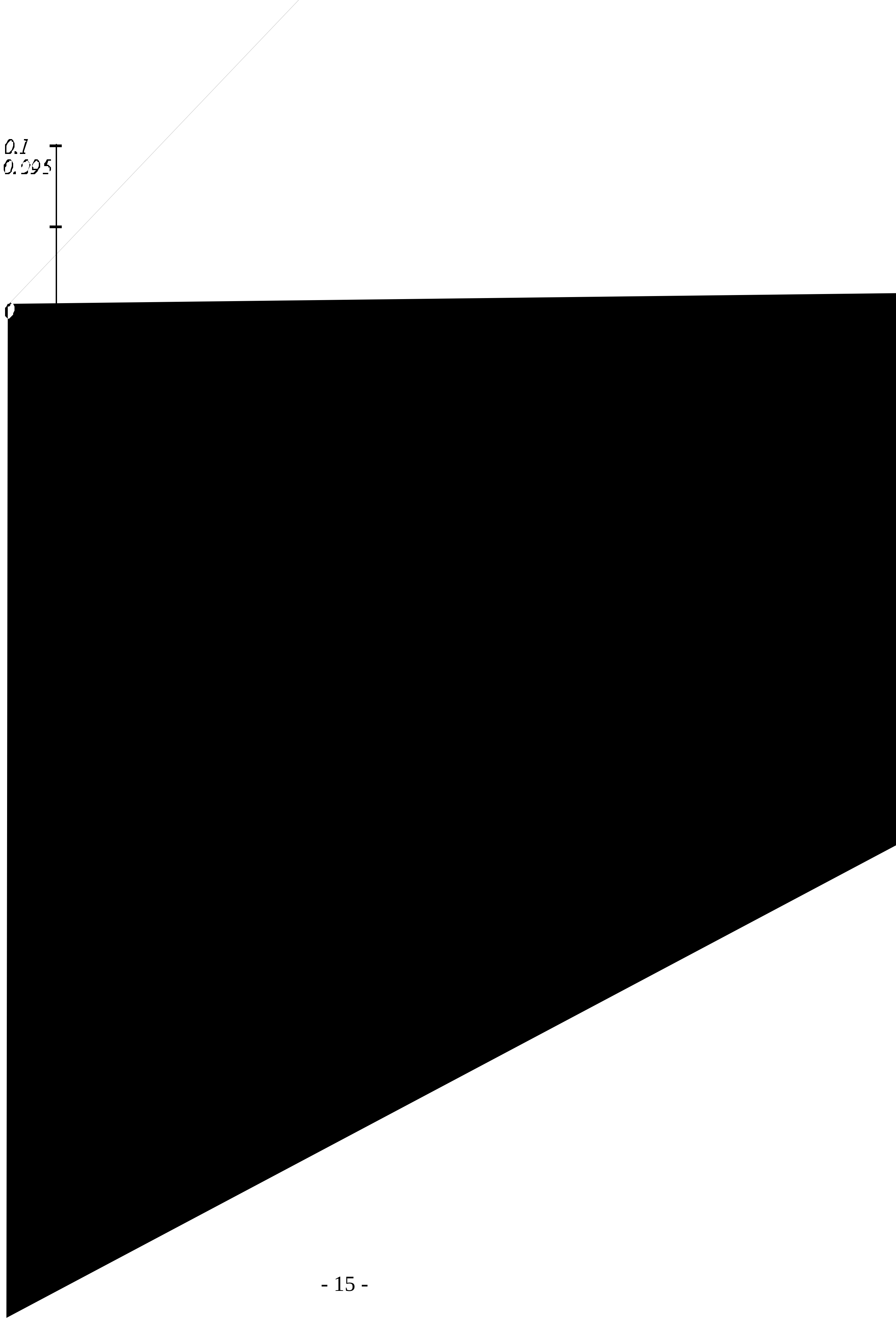
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## 3. INCOME MIXING BETWEEN COMMUNITIES

In this section we restrict attention to a possible second equilibrium with income mixing in which the urban area and the suburbs contain households of both income levels. Lemma A shows that the rich households live closer to the metropolitan center in each community. With both income classes living in both communities, the boundary between the rich and poor households occurs in the urban area at distance $x_u$ from the metropolitan center and in the suburbs at distance $y_s$ from the metropolitan center.

We use the structure developed in Section 2.[16] In particular, we continue to denote as $c_{ij}$ the total rent plus commuting cost of a household of income $M_i$ living in community $j$.[17] The reservation price of land is $r_0$

and that rich households receive equal utility in either community, or

$$U(M_2 - c_{2u} - g_u + T) + V(g_u) = U(M_2 - c_{2s} - g_s + T) + V(g_s) . \qquad (14)$$

As in Section 2, households vote taking the rent as given. If rich households were to form the majority in the urban area, rich households would prefer the urban area for both its commuting advantage and for its public service. Similarly, if poor households were to form the majority in both

$$U'(M_2 - c_{2s} - g_s + T) = V'(g_s). \tag{16}$$

Poor households live in the suburbs because the disadvantages of the high public service level, with the associated high tax, and the high commuting cost are exactly offset by the low rent. If there were undeveloped land in the urban area, housing in the urban area would be available at the reservation price $r_0$ so that poor households could move from the suburban fringe to the urban area without a change in rent. This would benefit them because they would obtain a lower commuting cost and their favored public service level. Hence at equilibrium the urban area must have no undeveloped land. This is formalized in Lemma D.

*LEMMA D: if rich and poor households live in both communities, there is no undeveloped urban land.*

PROOF: See Appendix B.

With no undeveloped urban land, equating supply and demand of land implies

$$\pi Y^2 = N\alpha; \tag{17}$$

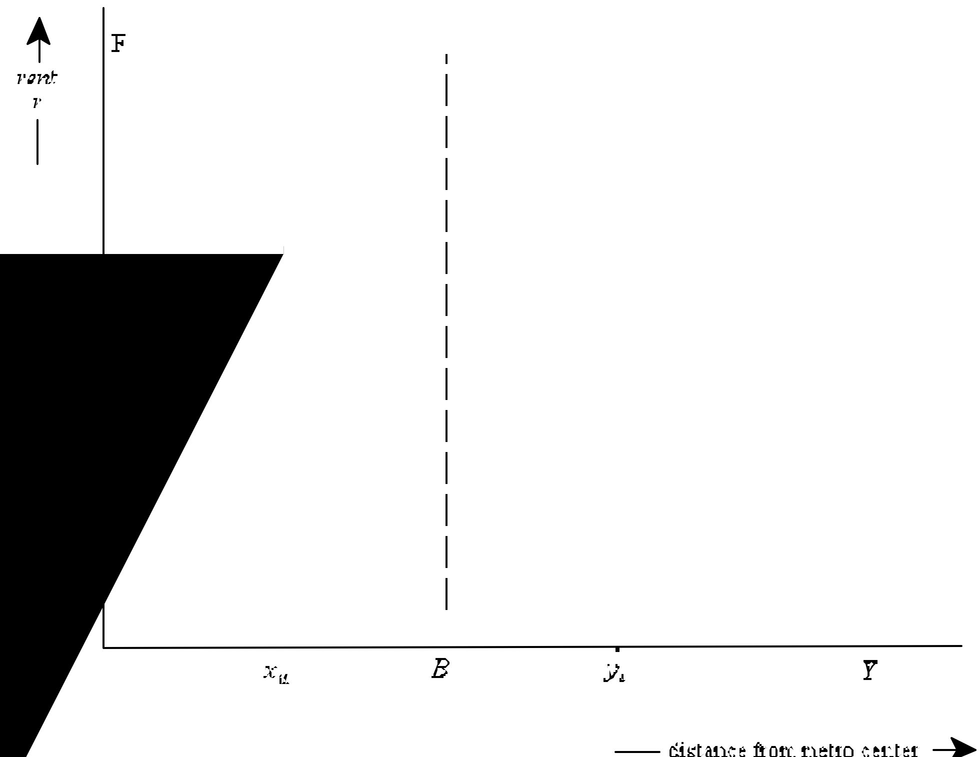
$$\pi x_u^2 + \pi \tag{18}$$

$$T = \frac{1}{N}\left[\int_0^{x_u} \frac{c_{2u} - tM_2 s}{a} 2\pi s\, ds + \int_{x_u}^{B} \frac{c_{1u} - tM_1 s}{a} 2\pi s\, ds \right.$$

$$\left. \int_{B}^{y_s} \frac{c_{2s} - tM_2 s}{a} 2\pi s\, ds + \int_{y_s}^{Y} \frac{c_{1s} - tM_1 s}{a} 2\pi s\, ds \right]. \qquad (19)$$

Note that $r_0 \$ 0$ implies $T > 0$.

The ten endogenous variables $c_{2u}$ , $c_{1u}$, $c_{2s}$ $c_{1s}$, $g_u$, $g_s$, $x_u$, $y_s$, $Y$ and $T$ are determined by the Equations (10)-(19).



metropolitan center, residents become rich and the rent gradient increases to $tM_2$. *ABG* is interpreted as the bid-rent curve of a poor household in the suburbs, and *BC* is interpreted as the bid-rent curve of a rich household in the suburbs.

As the location moves across the boundary between the suburbs and the urban area, the public service changes from the level set by rich households to the level set by poor households: poor households are willing to pay the premium *GD* to live in the urban area. *DE* is the bid-rent curve of a poor household in the urban area: the commuting advantage to the *poor* household of being closer to the metropolitan center is capitalized in rents, or rent rises along *DE* at rate $tM_1$

Rich households choose the suburban public service, so that rent at the boundary would have to fall by *CH* if a rich household were to be willing to live on the urban side of the boundary. *HEF* is the bid-rent curve of a rich household in the urban area. The vertical distance between *DE* and *a the bounda bound -30 TD -0*

$$\pi x_u^2 \le \frac{1}{2}\pi B^2 , \tag{20}$$

and rich households form the majority in the suburbs

$$\pi(y_s^2 - B^2) \ge \frac{1}{2}\pi(Y^2 - B^2) . \tag{21}$$

The ordering of distances must satisfy

$$0 < x_u < B < y_s < Y . \tag{22}$$

All consumption values must be non-negative, or[18]

$$M_1 - c_{1s} - g_s + T \ge 0 . \tag{23}$$

We now state the main result of this section.[19]

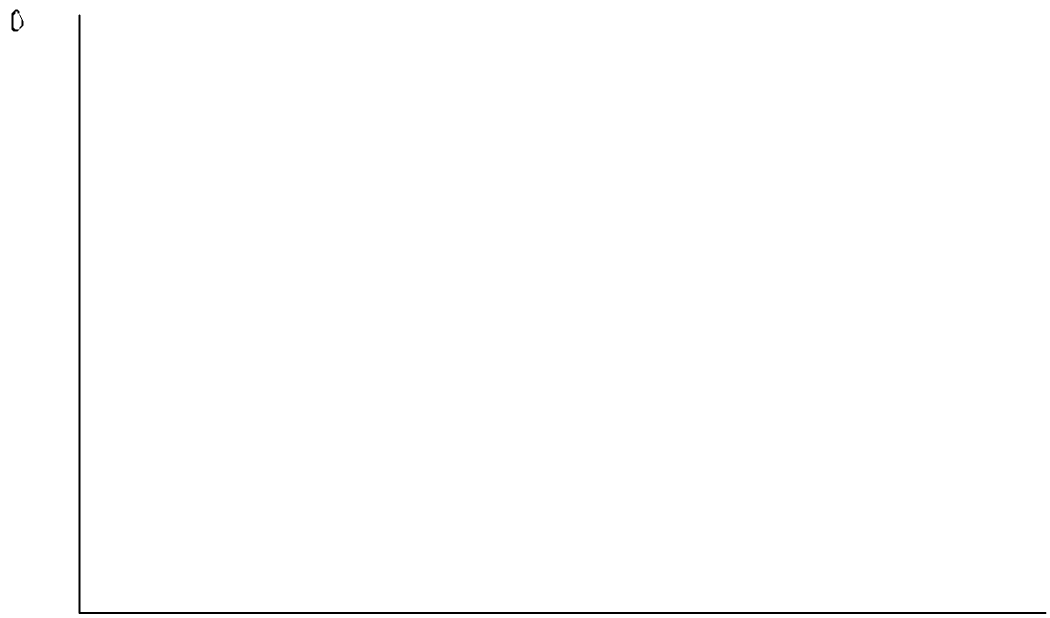
PROPOSITION 2: *There exist equilibria in which the poor and the rich households are located in both communities.*

PROOF: The proof is by construction of an example (see below).

and with parameter values:

We construct examples with utility having Cobb-Douglas form,

$$U = (1 - \alpha)\log_e(M - r(s) -$$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level of the suburbs (increasing ). Above the allowable region, commuting costs are sufficiently large that rich households migrate into the urban area and a required majority is reversed (Inequality (20) or (21) is violated). Below the allowable region, commuting costs are relatively unimportant so that rich households migrate into the suburban area to benefit from the higher public service and the urban area contains no rich households (Inequality (22) is violated) - a situation which descriptively corresponds to "urban flight".

Thirdly, Figure 5 shows that the region in which values of and $t$ support an equilibrium with income mixing between communities is "thick". Continuity implies that the equilibrium values change by only a small amount if there is a small change in the parameter values of the model. Therefore, provided the solution to Equations (10) - (19) lies strictly inside the allowable region defined by Inequalities (20)-(23), the equilibrium with both communities containing households of both income levels continues to exist if there is a small change in the parameter values of the model.

## 4. SUFFICIENT CONDITIONS

We now establish conditions which are sufficient to ensure the existence of an equilibrium with income mixing between communities. Consider the outcomes which would arise if the size of the rich suburban population were preset, and if rich households were immobile between communities but poor households could migrate between communities, i.e.,consider the solution to Equations (10)-(13) and (15)-(19) as a function of $y_s$. In particular, consider Allocation *A (B)* to be the allocation in which the rich suburban population has its lowest (highest) value consistent with income mixing between communities. If at Allocation *A (B)* rich suburban households achieve higher (lower) utility than rich urban households, then by continuity there is some rich suburban population at which rich households achieve equal utility in each community, or there is some $y_s$ at which Equation (14) is satisfied. At this outcome Inequalities (20), (21) and (22) are satisfied by construction. Additional restrictions need to be imposed to ensure positive consumption (Inequality 23).

Formally, we consider first the conditions which are sufficient to ensure that a solution to urban area contains no rich households or the suburbs contains no poor households,

$$y_s^B = \min y_s \; such\ that\ either \;\; x_u = 0 \;\; or \;\; y_s = Y.$$

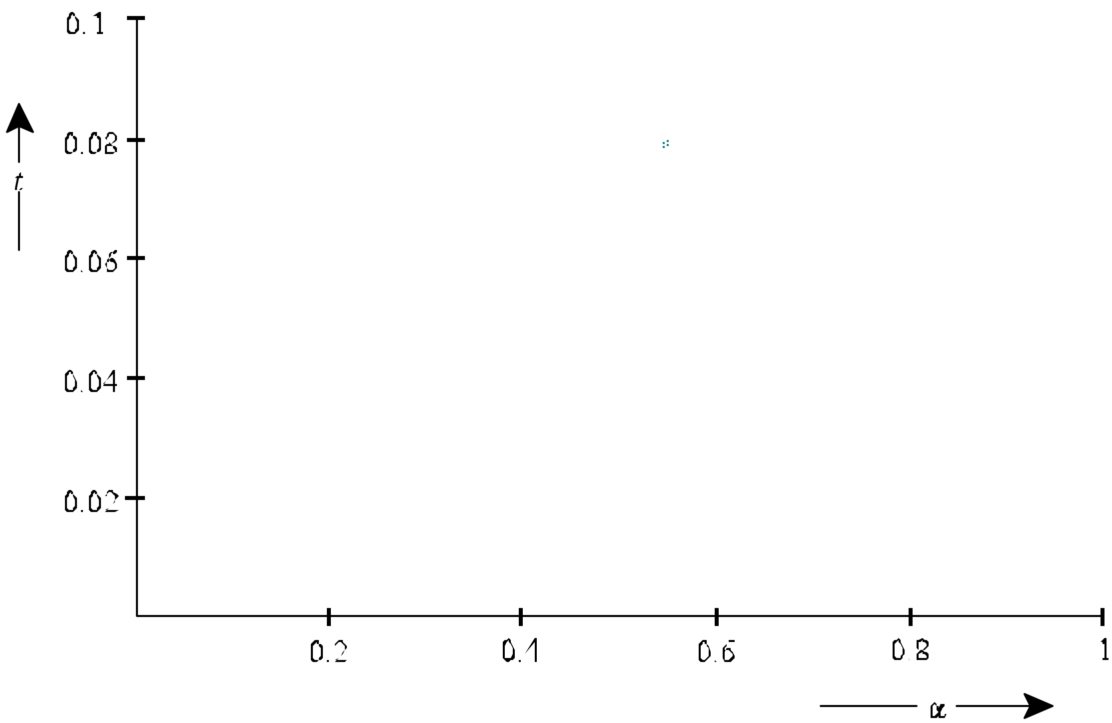
With $y_s = y_s^B$, denote the values of $c_{2u}, c_{1u}, c_{2s}, c_{1s}, g_u, g_s, x_u$ and $T$ which solve Equations

As $y_s$ increases from $y_s^A$ to $y_s^B$, a lower bound on $c_{2s}$ is

$$c_{2s}^* = r_0 + tM_1Y + t(M_2 - M_1)y_s^A .$$

## 4. WELFARE

In the first-best problem, community boundaries are flexible. The first-best efficient outcome has income sorting with the urban boundary set so that only rich households live in the urban area, $\pi B^2 = (1-\theta)N\alpha$. In this way, transportation costs are minimized and there is perfect matching of households with their desired public service level.



services. It is therefore not easy to make general statements.[20] Figure 7 shows two regions for the example used earlier, in which utility has Cobb-Douglas form,

$(1- \ ) \log_e c^h + \ \ \log_e g$ , and in which the parameter values are $M_1 = .3, M_2 = .6, \ \ = .6, B = 7,$

$Na = \ \ (10^2)$

urban rent must equal or exceed reservation value:

$$\max_{g} \left[ U(M_2 - c_{2u} - g + T) + V(g) \right] \geq U(M_2 - c_{1s} + tM_1X - tM_2$$

(Equation A.4). As above, $c_{1u} \leq c_{1t}$ and all variables change continuously so that *either* Case B1: there is

the assumption $M_1 - c_{1s} > 0$ implies that the proposed relationships $T(c_{2u})$ and $g_s(c_{2u})$ exist. Using Equations (A.14) and (A.16), if $c_{2u} = c_{2s}$,

$$c_{2u} = c_{2s} = c_{1s} + t(M_2 - M_1)y_s$$

$$= r_0 + tM_1Y + t$$

or

$$R\left(\frac{M_2+A}{1-\frac{X^2}{Y^2}}\right) = -\infty .$$

By continuity, there exists some $c_{2u}$

($M_1 > r_0 + tM_1\, Y + g_s$ ), it is therefore exactly “as if” there is a single community for which it is straightforward to show that an equilibrium exists.

PR [redacted] $s_A < s_B$. Suppose that

hous [redacted] cated at $s_B$. At equilibrium,

house [redacted]

a community, $s_A$ and $s_B$ with $s_A < s_B$. If $r(s_A) < r(s_B)$,

With $g_u = g_s$, this implies $c_{1u} = c_{1s}$. This contradicts Inequality (B.6).

With $c_{2u}, c_{1u}, c_{2s}, c_{1s}, g_u, g_s, x_u, Y$ and $T$ being continuous functions of $y_s$ (and $M_1$ being sufficient to ensure positive consumption), $S(y_s)$ is a continuous function of $y_s$ . By assumption, $S(y_s^A) > 0$ and $S(y_s^B) < 0$ . Hence, as $y_s$ changes from to , there must be at least one $y_s$ at which $S = 0$.

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1. Ross and Yinger (1999) review this literature.

2. If the income elasticities of land demand and of commuting costs are equal, the relationship between household income and distance from the metropolitan center is indeterminate. This is the case considered theoretically by Montesano (1972) and considered statistically relevant by Wheaton (1977).

3. Table 31990 Census of Population, Social and Economic Characteristics. Washington, D.C.: U.S. Department of Commerce.

4. Many cities have business districts dispersed throughout the metropolitan area in addition to the central business district. Our model of a circular metropolitan area and a central business district is therefore stylized. It is constructed to show how capitalization at different locations occurs at different rates and how this allows the income distribution in different communities to overlap. The logic can be extended to more complex spatial patterns.

5. For convenience of presentation, the public service is assumed to show constant returns to community population. Because each community contains a fixed number of households, no results change if the service is a local public good.

6. Without the fixed size assumption, demand for the public good would vary within an income class because housing price and income net of commuting cost vary over space.

7. Epple, Filimon, and Romer (1984) describe how such an infinite regress can arise with myopic voting.

8. Lemma A is a consequence of assuming that the income elasticity of land demand is zero and that per mile commuting costs increase with income. As noted in the Introduction, the pattern of sorting within a community is determined by comparing the income elasticity of land demand with the income elasticity of per mile transportation costs (Wheaton (1977)).

9. For a more general development, see Fujita (1989, Chap. 4).

10. Because $c_{ij}$ is a constant for all households of income $M_i$ living in community $j$, the analysis focuses on $c_{ij}$ and not on the rent schedule $r(s)$. However, the full rent schedule is:

$r(s: s=Y) = r_0$

12. This assumption is not important *per se*. What is important is that the two communities choose different public service levels. In addition, the proof of Proposition 1 requires that public service levels change continuously with $c_{2u}$ and $c_{1s}$.

13. It is simple to change the model to allow rents to be paid to absentee landlords.

14. This restriction is unnecessarily strong if $T > 0$ .

15. In this example the restriction $M_1 > r_0 + tM_1$ implies $t < .095$

been able to find this equilibrium with the Cobb-Douglas utilities. Note that this outcome is impossible if rents are not redistributed because one group is always either being moved to a smaller jurisdiction or experiences a decrease in the number of jurisdictions in which it is a majority. However, both groups may benefit from a better allocation if rents are redistributed.

22. Historically, the increase in metropolitan populations was accompanied by an improvement in transport technology lowering commuting costs. LeRoy and Sonstelie (1983) suggest how the history of advances in urban transportation might have led the observed outcome to be equilibrium with income mixing between communities.

23. Note that Equations (A.4) and (A.5) are obtained by rearranging the order of Equations (4a)-(4c) and (5a), (5b).

24. $U(M_2 - c_{2u} - g + T) + V(g)$ is strictly concave in $g$ so that the value of $g$ which maximizes $U(M_2 - c_{2u} - g + T) + V(g)$ is unique.

25. This implies his utility is higher in the urban area as the urban public service level is set to maximize his utility.

26. $U(M_2 - c_{2u} - g + T) + V(g)$ is strictly concave in $g$ so that the value of $g$ which maximizes $U(M_2 - c_{2u} - g + T) + V(g)$ is unique.

27. $g_s$ is being chosen to maximize $U(M_1 - c_{1s} - g + T) + V(g)$ where $U(c^h : c^h \# 0) = -4$ .

28. The requirement $x_u > 0$ reflects the discontinuity at $x_u = 0$.