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Search, Heterogeneity, and Optimal Income Taxation

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Search, Heterogeneity, and Optimal Income Taxation*

Working Paper

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Abstract

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D[illegible]. N[illegible], M[illegible] (1982), H[illegible] (1990) [illegible] A[illegible] (1999) [illegible]. H[illegible], [illegible] (2001) [illegible]. I[illegible].

[illegible]. I[illegible]. I[illegible] / [illegible] / [illegible]

2 Model

(H) (L)

l_k, $k = H; L$, q_m, $m = H; L$.

$y_{km} > 0$,

$y_{Hm} > y_{Lm}$.

$\in 0; 1$

$c_w(\ _k)$,

$c_w(0) = 0$, $c'_w(0) = 0$, $\delta_k \rightarrow$ $c'_w(\ _k) = +\infty$.

v_m, $c_\pi(v_m)$

2.1 The matching technology

, 1 – ff . A M P (1999) , . H , P (1996) Bl D -

2.2 Output sharing

2.4 Private expected utility functions

fi , , $v_H q_H =$ $_m q_m$.

k

$$U_k = -c_w(\ _k) + \ _k \ M(\ ) \ \frac{v_H q_H}{_m v_m q_m} \ _{kH} y_{kH} + \frac{v_L q_L}{_m v_m q_m} \ _{kL} y_{kL} \ + (1 - M(\ ))0 \ + (1 - \ _k)0$$

$$U_k = -c_w(\ _k) + \ _k M(\ ) E_{(m)} \ _{km} y_{km}; \tag{5}$$

$E_{(m)}$. A , , $c(\ )$, . O , , $M(\ )$, . I , $1 - M(\ )$, , , . I ,

(

3 Optimal search intensity and market ine ciencies

3.1 Social Optimum

A

$$W = {}_{\delta,v} \quad {}_{k}\, l_k U^k + {}_{m}\, q_m V^m$$

$$. \quad . \quad {}_{k} \geq 0; \quad v_m \geq 0:$$

(1), (2), (5), (6), -91.21

$$E_{(m)}y_{km} - (1 - \quad)E_{(k)}E_{(m)}y_{km} = \quad E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}:$$

A ... (1 ...) ... $E_{(k)}E_{(m)}y_{km}$), ... $E_{(m)}y_{km}$, ... $E_{(k)}E_{(m)}y_{km}$. ...

... :

... 1. I ...

$$c'_{\pi}(\bar{v}_H) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kH} - \quad E_{(k)}E_{(m)}y_{km}, \quad c'_{\pi}(\bar{v}_L) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kL} - \quad E_{(k)}E_{(m)}y_{km} \quad \Bigg| \quad \bar{\ } \lesseqgtr 1; \ \bar{v}_H > 0; \ \bar{v}_L > 0 \quad ; \tag{13}$$

$$c'_{\pi}(\bar{v}_H) = \frac{M(\ )}{} E_{(k)}y_{kH} - \quad E_{(k)}E_{(m)}y_{km}, \quad c'_{\pi}(0) \geq \frac{M(\ )}{} E_{(k)}y_{kL} - \quad E_{(k)}E_{(m)}y_{km} \quad \Bigg| \quad \bar{\ } \lesseqgtr 1; \ \bar{v}_H > 0; \ \bar{v}_L = 0 \quad : \tag{14}$$

3.2 Decentralized equilibrium

A ... k ...

$$\max_{\delta_k} \ U_k = -c_w(\ _k) + \ _k M(\ )E_{(m)} \ _{km}y_{km} \quad \text{s. t.} \ _k \geq 0; \tag{15}$$

... $C_k = c_w(\ _k)$ ... , ... $B_k = \ _k M(\ )E_{(m)} \ _{km}y_{km}$ ...

K

$$-c'_w(\ _k) + M(\ )E_{(m)}\ _{km}y_{km} \le 0$$

$$_k \ge 0 \qquad (16)$$

$$(-c'_w(\ _k) + M(\ )E_{(m)}\ _{km}y_{km})\ _k = 0;$$

$M(\ )$, … $_{LL} \neq 0$ ($_{LH} = 0$), … $y_{km} > 0$,

(06)2 158.730 ( ) -395. 4.9812163(0),)-373( )-333(( )-350(5. 4

$_H M(\ )E_{(m)} w_{Hm}$.

$\left({}_L \frac{E_{(m)} w_{Lm}}{E_{(m)} w_{Hm}} \right.$ $M(\ )E_{(m)} w_{Hm} =$ $_L M(\ )E_{(m)} w_{Lm}$, $\left({}_L \frac{E_{(m)} w_{Lm}}{E_{(m)} w_{Hm}} \right.$

$_L M(\ )E_{(m)} w_{Lm}$ [3].

$_H M(\ )E_{(m)} w_{Hm}$ $_L M(\ )E_{(m)} w_{Lm}$, ^w_H, $_H M(\ )E_{(m)} w_{Hm}$, ^w_L, $_L M(\ )E_{(m)} w_{Lm}$.

[13]

: -

. D ll K (1992), ll B B (2002),

, fi

ff ll ff ;

fi . fi ,

. O fi

4. *I*

$$c'_w(\ _k) = M(\ )(1 - \ ^w_k)w_k \qquad \lesseqgtr 1;$$

$$c'_\pi(v_m) = \frac{M(\ )}{\ }(1 - \ ^\pi_m)\ _m \qquad _k > 0;\ v_m > 0 \quad ; \tag{22}$$

$$c'_w(0) \geq M(\ )(1 - \ ^w_L)w_L \qquad \lesseqgtr 1;$$

$$c'_\pi(0) \geq \frac{M(\ )}{\ }(1 - \ ^\pi_L)\ _L \qquad _L = 0;\ v_L = 0 \quad : \tag{23}$$

4.1 Characterizing externalities through Pigou taxes

, P $(\check{\ }^w_H; \check{\ }^w_L)$, P $(\check{\ }^\pi_H; \check{\ }^\pi_L)$. L ( ) $\check{R}$, P , , LS:

$$\check{R} = (\ _k\ l)M(\ )\ \frac{_H l_H}{_k\ l}\check{\ }^w_H w_H + \frac{_L l_L}{_k\ l}\check{\ }^w_L w_L + \frac{v_H q_H}{_m vq}\check{\ }^\pi_H\ _H + \frac{v_L q_L}{_m vq}\check{\ }^\pi_L\ _L$$

$$0 = \check{R} - \sum_k l_k + \sum_m l_m\ \ LS;$$

$(\ _k\ l)M(\ ) = N$.

k

$$U_k = -c_w\ \frac{z^w_k}{M(\ )\,w_k} + LS + (1 - \check{\ }^w_k)z^w_k \tag{24}$$

, LS, ff

. fi ,

c'_w

[illegible] ([illegible] P [illegible] 5) [illegible]. CR [illegible], [illegible]; [illegible], [illegible]. [illegible], P [illegible], [illegible]. [illegible], [illegible] ff [illegible]. I[illegible] fi[illegible], [illegible] fi[illegible], L [illegible], [illegible].

N [illegible], [illegible] ff [illegible] $(g = 0)$. [illegible], [illegible]. I[illegible]

$y_{LL} > 0$, ...

4.2 Optimal income taxes with positive government revenue

... R, ...

$$W = \quad l_k U^k + \quad q_m V^m \quad ;$$

(1), (2), (5), (6), ... $w_k = E_{(m)} \quad {}_{km} y_{km}$, $z^w_k = \quad {}_k M(\ ) w_k$, ${}_m = E_{(k)}(1 - \quad {}_{km}) y_{km}$, $z^\pi_m = v_m \frac{M\ \theta}{\theta} \quad {}_m$ ... 4, ...

$$W = \quad l_k \quad -c_w \quad \frac{z^w_k}{M(\ )\, w_k} \quad + \quad q_m \quad -c_\pi \quad \frac{z^\pi_m}{\frac{M\ \theta}{\theta} \quad {}_m} \quad + (\quad {}_k \quad l) M(\ )\, E_{(k)} E_{(m)} y_{km};$$

$(\quad {}_k \quad l) M(\ ) = N$ ...

R, ...

$$R \leq (\quad {}_k \quad l) M(\ ) \quad \frac{{}_H l_H}{{}_k \quad l} \ {}^w_H w_H + \frac{{}_L l_L}{{}_k \quad l} \ {}^w_L w_L + \frac{v_H q_H}{{}_m \ vq} \ {}^\pi_H \quad {}_H + \frac{v_L q_L}{{}_m \ vq} \ {}^\pi_L \quad {}_L \quad ; \qquad (30)$$

$(\quad {}_k \quad l) M(\ ) = M$ ... (30), ...

17 ...

$$W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\ ) w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{\frac{M\ \theta}{\theta}\ m} \right)$$

$$+ \left(\sum_k l\right) M(\ ) \left[\frac{{}_H l_H}{\sum_k l}(1 - {}^w_H) w_H + \frac{{}_L l_L}{\sum_k l}(1 - {}^w_L) w_L \right]$$

$$+ \frac{v_H q_H}{\sum_m vq}(1 - {}^\pi_H)\ {}_H + \frac{v_L q_L}{\sum_m vq}(1 - {}^\pi_L)\ {}_L + R:$$

... (22) (... C ... (1978), ... D ... (1998) ...).

H ...

L ... 7 ... A ... E ... (22) ...

$$\max_{\tau_k^w, \tau_m^\pi} W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\ ) w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{M\ \theta} \right)$$

P 8 A . -

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. I ,

. I ,

.

ff ,

$''^{w}_{k}=0$ $''^{\pi}_{m}=0$. I . I

, , , (33).

. H , ff

. I ,

, ,

(P 5 A A). H ,

- ,

.

I fi , $''^{\pi}_{m}=\infty$, ,

, .

I ' .

$''^{\pi}_{m}=\infty$,

. , ,

ff .

4.2.2

I I

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1

. A

P 5,

: fi

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; , .

12.

$$i)\ @ \frac{w_H}{w_L} = @ \frac{E_{(m)} \quad Hm}{E_{(m)} \quad Lm} < 0 \qquad (36)$$

$$ii)\ @ \frac{w}{\pi} = @(\ ) < 0 \qquad (37)$$

P 11 A . B

, (

P 5).

( ), .

, .

P 12

, .

B B (2002)

5 Conclusion

, -

. I

(1) "H Effi F ," I E , 40, 827-849.

(2002) "O L ," J E , 85, 53-97.

(1 4) "Eff B fi F ," P .

(1) "M Cl ," J E , 112, 141-168.

(1) "L - P F : M E ," E J , 109, 307-334.

(1) "O R : M R ," A E , 68, 756-768.

(1 1) "M C , F Effi ," J E , .89, 798-812.

(1 2) "A D M E ," J E , .90, 881-894.

(1 2) " D Effi E ," E , .49, 217-227.

(1) "O I : A E P O M R ," A E , .88, 83-95.

(1) " P ," E J , 7, 46-70.

(1 3) "O O P I ," J E , 2, 357-376.

(1) "H D P Aff E ? C O E ," M P .

. (1 0) "Effi I ," *J E* , 51, 8999.

(1 5) "P , , P , D E , .

. (1 0) "O Effi M R M ," *E* , 57, 279-298.

. (1) " , F , ," *E* , 32, D E , .

(1) " P G E P M B ," *L E* , 3, 6580.

. (1) "L R , G J B J ," *E* , *C* , P .

(1 3) "

., (1) "L M D Eff , " J E , 33, 457495.

, ., (2000) "A M , " E , 68, 343-369.

, ., (2001) "M , , H " A , 1(2001), I . 1, A . 5.

. (1) "P F E P ?," E , 25, 223264.

. (1) "O P F L M ," L E , 6, 435452.

, . (1) "P Effi O N N E ," I : H E , .2.

, . (1) " - P Effi ," J E , 6, 239262.

, . (1) " , P P D ." I F , 6, 239262.

, . (1) " P F E ," I : , .B. (E .), F C ., M M , L .

Appendices:

A Proofs of the main results

Proof of Corollary 3.

F , . - , $_1 \neq$. I $_1 >$, $_H(_1) > _H()$, $_L(_1) > _L()$, $v_H(_1) < v_H()$, $v_L(_1) < v_L()$. H))

CR

F 1 − =

$$\check{R} = N \left[\begin{array}{l} \frac{\delta_H l_H}{k\,\delta l} \; (1 - \;\;) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \;\; {}_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\,\delta l} \; (1 - \;\;) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \;\; {}_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\, vq} \; (1 - (1 - \;\;)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \;\; {}_{kH} y_{kH} \\ + \frac{v_L q_L}{m\, vq} \; (1 - (1 - \;\;)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \;\; {}_{kL} y_{kL} \end{array} \right]$$

$$= N \left[\begin{array}{l} \frac{\delta_H l_H}{k\,\delta l} \; (1 - \;\;) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \;\; {}_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\,\delta l} \; (1 - \;\;) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \;\; {}_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\, vq} \; (1 - \;\;) E_{(k)} E_{(m)} y_{km} - E_{(k)} \;\; {}_{kH} y_{kH} \\ + \frac{v_L q_L}{m\, vq} \; (1 - \;\;) E_{(k)} E_{(m)} y_{km} - E_{(k)} \;\; {}_{kL} y_{kL} \end{array} \right]$$

$$\check{R} = N \;\; 1 - (\;\; + \;\;)$$

; - , . I (L).□

Proof of Lemma 7.

I , k

$$U_k = \; - c_w (\;\; {}_k) + \;\; {}_k M(\;\;)(1 - \;\; {}^w_k) w_k$$

$$= \; - c_w \;\; \frac{z^w_k}{M(\;\;) w_k} \;\; + (1 - \;\; {}^w_k) z^w_k$$

w_k

$$\frac{@U_k}{@w_k} = -c'_w \frac{1}{M(\;\;) w_k} \frac{@z^w_k}{@w_k} - c'_w \frac{z^w_k}{M(\;\;)} \;\; -\frac{1}{w_k^2} \;\; + \frac{@z^w_k}{@w_k} (1 - \;\; {}^w_k) = c'_w \frac{z^w_k}{M(\;\;) w_k} > 0;$$

E . □

Proof of Proposition 8.

I $w_k = E_{(m)} w_{km} = E_{(m)} \quad _{km} y_{km}$

$k = H; L;$ $z_k^w = \quad _k M(\quad) w_k$

$k = H; L;$ $\quad _m = E_{(k)} \quad _{km} = E_{(k)} (1 - \quad _{km}) y_{km}$ fi

e $m = H; L;$ $z_m^\pi = v_m$ M

Z π π

k

[illegible]

$$\frac{dz_H^w}{z_H^w}\left(\frac{1}{''^w_H} + (1- \ )\frac{{}_H l_H}{k \ l}\right) = (1- \ )\ E_{(m)}\left(\frac{dz_m^\pi}{z_m^\pi} - \frac{{}_L l_L}{k \ l}\frac{dz_L^w}{z_L^w}\right) - \frac{d \ {}^w_H}{1- \ {}^\pi} \qquad ($$

[illegible]

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{''^\pi_H} = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad + \frac{d\tau_L^\pi}{-\tau_L^\pi} \quad \frac{v_L q_L}{_m\, vq}''^\pi_L - \frac{d\tau_H^\pi}{-\tau_H^\pi}\Big(1 + \quad \frac{v_L q_L}{_m\, vq}''^\pi_L}{\Delta_2} \tag{45}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{''^\pi_L} = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad - \frac{d\tau_L^\pi}{-\tau_L^\pi}\Big(1 + \quad \frac{v_H q_H}{_m\, vq}''^\pi_H \quad + \frac{d\tau_H^\pi}{-\tau_H^\pi} \quad \frac{v_H q_H}{_m\, vq}''^\pi_H}{\Delta_2}; \tag{46}$$

[illegible] $\Delta_2 = 1 + \quad E_{(m)}''^\pi_m$. A [illegible] (45) [illegible] (46) [illegible]

$$E_{(m)} \quad \frac{dz_m^\pi}{z_m^\pi} \quad = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad E_{(m)}''^\pi_m - E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2}; \tag{47}$$

[illegible] (43) [illegible] (44) [illegible]

$$E_{(k)} \quad \frac{dz_k^w}{z_k^w} \quad = \frac{(1- \quad)E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi} \quad E_{(k)}''^w_k - E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w}}{\Delta_1}: \tag{48}$$

F [illegible] (47) [illegible] (48) [illegible] $E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi}$ [illegible] $E_{(k)}\Big(\frac{dz_k^w}{z_k^w}$ [illegible]

[illegible]

$$E_{(m)} \quad \frac{dz_m^\pi}{z_m^\pi} \quad = - \frac{(\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1 + \Delta_2 - 1} \tag{49}$$

$$E_{(k)} \quad \frac{dz_k^w}{z_k^w} \quad = - \frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1 + \Delta_2 - 1} \tag{50}$$

[illegible] (49) [illegible] (43) [illegible] (44), [illegible] (50) [illegible] (45) [illegible] (46) [illegible]

[illegible]:

$$\frac{dz_H^w}{z_H^w}\frac{1}{''^w_H} = - \frac{(1- \quad) \quad (\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$
$$+ \frac{(\Delta_1 + \Delta_2 - 1) \quad \frac{d\tau_L^w}{-\tau_L^w}(1- \quad)\frac{\delta_L l_L}{_k\, \delta l}''^w_L - \frac{d\tau_H^w}{-\tau_H^w}\Big(1 + (1- \quad)\frac{\delta_L l_L}{_k\, \delta l}''^w_L}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{51}$$

$$\frac{dz_L^w}{z_L^w}\frac{1}{''^w_L} = - \frac{(1- \quad) \quad (\Delta_2 - 1)E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$
$$+ \frac{(\Delta_1 + \Delta_2 - 1) \quad - \frac{d\tau_L^w}{-\tau_L^w}\Big(1 + (1- \quad)\frac{\delta_H l_H}{_k\, \delta l}''^w_H \quad + \frac{d\tau_H^w}{-\tau_H^w}(1- \quad)\frac{\delta_H l_H}{_k\, \delta l}''^w_H}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{52}$$

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{''^\pi_H} = -\frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1-1)E_{(m)}\Big(''^\pi_m\frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)} + \frac{(\Delta_1+\Delta_2-1)\quad \frac{d\tau_L^\pi}{-\tau_L^\pi}\quad \frac{v_L q_L}{_k vq} ''^\pi_L - \frac{d\tau_H^\pi}{-\tau_H^\pi}\Big(1+ \quad \frac{v_Lq_L}{_k vq}''^\pi_L}{\Delta_2(\Delta_1+\Delta_2-1)}; \quad (53)$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{''^\pi_L} = -\frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1-1)E_{(m)}\Big(''^\pi_m\frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)} + \frac{(\Delta_1+\Delta_2-1)\quad -\frac{d\tau_L^\pi}{-\tau_L^\pi}\Big(1+ \quad \frac{v_H q_H}{_k vq} ''^\pi_H \quad + \frac{d\tau_H^\pi}{-\tau_H^\pi}\quad \frac{v_Hq_H}{_k vq}''^\pi_H}{\Delta_2(\Delta_1+\Delta_2-1)}; \quad (54)$$

F [illegible] (51)-(54) [illegible] fi [illegible].

$$\begin{aligned}
\frac{dz_H^w}{d\ _H^w}\frac{1}{z_H^w} &= \frac{''^w_H}{1-\ _H^w}\quad\frac{(1-\quad)\frac{\delta_H l_H}{_k\delta l}''^w_H - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ _H^w}\frac{1}{z_L^w} &= \frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\delta l}''^w_L(1-\quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ _H^w}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\delta l}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ _H^w}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\delta l}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \quad (55)$$

$$\begin{aligned}
\frac{dz_H^w}{d\ _L^w}\frac{1}{z_H^w} &= \frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\delta l}''^w_H(1-\quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ _L^w}\frac{1}{z_L^w} &= \frac{''^w_L}{1-\ _L^w}\quad\frac{(1-\quad)\frac{\delta_L l_L}{_k\delta l}''^w_L - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ _L^w}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\delta l}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ _L^w}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\delta l}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \quad (56)$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}^{\pi}_{H}}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\,''^{w}_{H}(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}^{\pi}_{H}}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\,''^{w}_{L}(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}^{\pi}_{H}}\frac{1}{z_H^\pi} &= \frac{''^{\pi}_{H}}{1-\ {}^{\pi}_{H}}\quad\frac{''^{\pi}_{H}\frac{v_H q_H}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}^{\pi}_{H}}\frac{1}{z_L^\pi} &= \frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\,''^{\pi}_{L}}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}^{\pi}_{L}}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\,''^{w}_{H}(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}^{\pi}_{L}}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\,''^{w}_{L}(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}^{\pi}_{L}}\frac{1}{z_H^\pi} &= \frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\,''^{\pi}_{H}}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}^{\pi}_{L}}\frac{1}{z_L^\pi} &= \frac{''^{\pi}_{L}}{1-\ {}^{\pi}_{L}}\quad\frac{''^{\pi}_{L}\frac{v_L q_L}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{58}
$$

[illegible]

[illegible]

$$
\begin{aligned}
{}_{\tau_k^w,\tau_m^\pi} W = &\ \sum_k l_k \ -c_w\ \frac{z_k^w}{M(\ )\,w_k} \ + \sum_m q_m\ -c_\pi\ \frac{z_m^\pi}{\frac{M\,\theta}{\theta}\ {}_m}\\
&+ {}_H l_H c_w'\ \frac{z_H^w}{M(\ )\,w_H} + {}_L l_L c_w'\ \frac{z_L^w}{M(\ )\,w_L} + v_H q_H c_\pi'\ \frac{z_H^\pi}{\frac{M\,\theta}{\theta}\ {}_H} + v_L q_L c_\pi'\ \frac{z_L^\pi}{\frac{M\,\theta}{\theta}\ {}_L} + R\\
&+ (\ {}_k\ l)M(\ )\ \frac{{}_H l_H}{{}_k\ l}\ {}^{w}_{H} w_H + \frac{{}_L l_L}{{}_k\ l}\ {}^{w}_{L} w_L + \frac{v_H q_H}{{}_m vq}\ {}^{\pi}_{H}\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}^{\pi}_{L}\ {}_L\ ;
\end{aligned}
$$

[illegible]

$$
a = \frac{{}_H l_H}{{}_k\ l}\ {}^{w}_{H} w_H + \frac{{}_L l_L}{{}_k\ l}\ {}^{w}_{L} w_L \qquad b = \frac{v_H q_H}{{}_m vq}\ {}^{\pi}_{H}\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}^{\pi}_{L}\ {}_L :
$$

$$\frac{\partial L}{\partial \tau_H^w} =$$

$$= \sum_k l_k \left(-\frac{c_w'}{M(\theta)\, w_k}\frac{dz_k^w}{d\tau_H^w}\right) + \sum_m q_m \left(-\frac{c_\pi'}{\frac{M(\theta)}{\theta}\pi_m}\frac{dz_m^\pi}{d\tau_H^w}\right)$$

$$+\frac{dz_H^w}{d\tau_H^w}\frac{1}{M(\theta)\, w_H} l_H c_w' \left(\frac{z_H^w}{M(\theta)\, w_H}\right) + \nu_H l_H c_w'' \left(\frac{z_H^w}{M(\theta)\, w_H}\right)\frac{1}{M(\theta)\, w_H}\frac{dz_H^w}{d\tau_H^w}$$

$$+\frac{dz_L^w}{d\tau_H^w}\frac{1}{M(\theta)\, w_L} l_L c_w' \left(\frac{z_L^w}{M(\theta)\, w_L}\right) + \nu_L l_L c_w'' \left(\frac{z_L^w}{M(\theta)\, w_L}\right)\frac{1}{M(\theta)\, w_L}\frac{dz_L^w}{d\tau_H^w}$$

$$+\frac{dz_H^\pi}{d\tau_H^\pi}\frac{1}{\frac{M(\theta)}{\theta}\pi_H} q_H c_\pi' \left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\pi_H}\right) + \nu_H q_H c_\pi'' \left(\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\pi_H}\right)\frac{1}{\frac{M(\theta)}{\theta}\pi_H}\frac{dz_H^\pi}{d\tau_H^\pi}$$

$$+\frac{dz_L^\pi}{d\tau_L^\pi}\frac{1}{\frac{M(\theta)}{\theta}\pi_L} q_L c_\pi' \left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\pi_L}\right) + \nu_L q_L c_\pi'' \left(\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\pi_L}\right)\frac{1}{\frac{M(\theta)}{\theta}\pi_L}\frac{dz_L^\pi}{d\tau_L^\pi}$$

$$+\left[\sum_k \frac{l_k}{M(\theta)\, w_k}\frac{dz_k^w}{d\tau_H^w} M(\theta) + \left(\sum_k l\right) M(\theta)\left(\frac{\sum_m \frac{q_m}{\frac{M(\theta)}{\theta}\pi_m}\frac{dz_m^\pi}{d\tau_H^w}}{\sum_k l} - \frac{\left(\sum_m \nu q\right)\sum_k\left(\frac{l_k}{M(\theta)\, w_k}\frac{dz_k^w}{d\tau_H^w}\right)}{\left(\sum_k l\right)^2}\right)\right](a+b)$$

[illegible]

$$+\left(\sum_k l\right) M(\theta)$$

fi

$$ {}_H l_H\, c''_w \; \frac{z^w_H}{M(\;)\, w_H} \quad \frac{1}{M(\;)\, w_H} \frac{dz^w_H}{d\;{}^w_H} + \;{}_L l_L\, c''_w \; \frac{z^w_L}{M(\;)\, w} $$ [redacted]

$$ + v_H q_H\, c''_\pi \; \frac{z^\pi_H}{\frac{M\,\theta}{\theta}\;{}_H} \quad \frac{1}{\frac{M\,\theta}{\theta}\;{}_H} \frac{dz^\pi_H}{d\;{}^w_H} + v_L q_L\, c''_\pi \; \frac{z^\pi_L}{\frac{M\,\theta}{\theta}\;{}_L} $$ [redacted]

$$ + \;(\;\;{}_k\;\; l) M(\;) \Bigg[\frac{{}_k\left(\frac{l_k}{M\,\theta\;\; w_k}\frac{dz^w_k}{d\tau^w_H}\right.}{{}_k\;\; l} + \frac{M'(\;)}{M(\;)} \frac{m}{} $$ [redacted] $$ \frac{\frac{l_k}{M\,\theta\;\; w_k}\frac{dz^w_k}{d\tau^w_H}}{{}_k\;\; l} \Bigg] \Big| \;(a+b) = $$

$$ = \;{}_H l_H\, c''_w \; \frac{z^w_H}{M(\;)\, w_H} \quad \frac{1}{M(\;)\, w_H} \frac{dz^w_H}{d\;{}^w_H} \frac{z^w_H}{z^w_H} \frac{M(\;)\,(}{c'_w(z^w_H =} $$ [redacted]

$$ + \;{}_L l_L\, c''_w \; \frac{z^w_L}{M(\;)\, w_L} \quad \frac{1}{M(\;)\, w_L} \frac{dz^w_L}{d\;{}^w_H} \frac{z^w_L}{z^w_L} \frac{M(\;)\,(1 - \;{}^w_L) w_L}{c'_w(z^w_L = M(\;)\, w_L)} $$

$$ + v_H q_H\, c''_\pi \; \frac{z^\pi_H}{\frac{M\,\theta}{\theta}\;{}_H} \quad \frac{1}{\frac{M\,\theta}{\theta}\;{}_H} \frac{dz^\pi_H}{d\;{}^w_H} \frac{z^\pi_H}{z^\pi_H} \frac{M(\;)}{} \frac{(1 - \;{}^\pi_H)\;\;{}_H}{c'_\pi(z^\pi_H = \;\;{}_H)} $$

$$ + v_L q_L\, c''_\pi \; \frac{z^\pi_L}{\frac{M\,\theta}{\theta}\;{}_L} \quad \frac{1}{\frac{M\,\theta}{\theta}\;{}_L} \frac{dz^\pi_L}{d\;{}^w_H} \frac{z^\pi_L}{z^\pi_L} \frac{M(\;)}{} \frac{(1 - \;{}^\pi_L)\;\;{}_L}{c'_\pi\left(z^\pi_L = \frac{M\,\theta}{\theta}\;\;{}_L\right.} $$

$$ + \;(\;\;{}_k\;\; l) M(\;) \Bigg[\;\; {}_k\left(\frac{l_k}{M\,\theta\;\; u\mathrm{wH}}\right. $$

$$= \ _{H} l_{H} \frac{1}{''^{w}_{H}} \frac{dz^{w}_{H}}{d \ ^{w}_{H}} \frac{1}{z^{w}_{H}}$$

${}^{\pi}_{H}$, ${}^{\pi}_{L}$

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad)^{1 - \quad {}^{w}_{L}}$$

F

$$
= 1 - \frac{\begin{bmatrix} \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\, vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\, vq}\right) w_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\, vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\, vq}\right) w_{HL} \quad {}^{w}_{H} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\, vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\, vq}\right) w_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\, vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\, vq}\right) w_{LL} \quad {}^{w}_{L} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\, vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\, vq}\right) {}_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\, vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\, vq}\right) {}_{HL} \quad {}^{\pi}_{H} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\, vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\, vq}\right) {}_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\, vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\, vq}\right) {}_{LL} \quad {}^{\pi}_{L} \end{bmatrix}}{\frac{-\tau^w_H}{\varepsilon^w_H} w_H + \frac{-\tau^w_L}{\varepsilon^w_L} w_L + \frac{-\tau^\pi_H}{\varepsilon^\pi_H} {}_H + \frac{-\tau^\pi_L}{\varepsilon^\pi_L} {}_L} ;
$$

$$
= \left(1 - \frac{E_{(s)} w_{Hs} \, {}^{w}_{H} + E_{(s)} w_{Ls} \, {}^{w}_{L} + E_{(s)} \, {}_{sH} \, {}^{\pi}_{H} + E_{(s)} \, {}_{sL} \, {}^{\pi}_{L}}{\frac{-\tau^w_H}{\varepsilon^w_H} w_H + \frac{-\tau^w_L}{\varepsilon^w_L} w_L + \frac{-\tau^\pi_H}{\varepsilon^\pi_H} {}_H + \frac{-\tau^\pi_L}{\varepsilon^\pi_L} {}_L}\right)^{-} ; \qquad (64)
$$

s $\left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\, vq} + \ldots\right.$

c'

$$'' = \frac{1}{\ } / \frac{c''}{c'} = \frac{1}{\ -1}.$$

A,

.

2 $c = A(\ ^{\gamma} + \ ^{\beta})$ ≥ 3, $<$, $c' = A(\ ^{\gamma-} + \ ^{\beta-})$,

$$c'' = A(\ (\ -1)\ ^{\gamma-2} + \ (\ -1)\ ^{\beta-2}) > 0;$$

$$'' = \frac{\ ^{\gamma-2} + \ ^{\beta-2}}{(\ -1)\ ^{\gamma-2} + (\ -1)\ ^{\beta-2}}.$$

A , , .

$$\frac{\partial ''}{\partial} = -\ ^{\beta}\ ^{\gamma-}\ (\ -\ )^2 < 0.$$

N ≥ 3 $= \ -1$. . F ,

< 1 $= 3$ $= 2$, > 1 $= 2$ $= 1$.

$\frac{-\tau_H^w}{\varepsilon_H^w} w_H < \frac{-\tau_L^w}{\varepsilon_L^w} w_L$, (66) -

. H $(1 - \ _H^w) w_H < (1 - \ _L^w) w_L$ $\ _H^w > \ _L^w$

(66) $\ _H^w w_H > \ _L^w w_L$. N $(E_{(m)}\ _{Hm}\ _m^{\pi} - E_{(m)}\ _{Lm}\ _m^{\pi})$. -

$\ _{HH} > \ _{LH}$ $\ _{HL} > \ _{LL}$ $(E_{(m)}\ _{Hm}\ _m^{\pi} - E_{(m)}\ _{Lm}\ _m^{\pi} > 0)$.

H (66) .

$(1 - \ _H^w) w_H > (1 - \ _L^w) w_L$, $\ _H > \ _L$, $''^w_H \leq ''^w_L$. □

Proof of Proposition 11.

fi (65).

fi

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(60)-(63)

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, . D

(60) (61)

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad) \frac{1 - \quad ^{w}}{''_{w}} w + \quad (w \quad ^{w} + \quad ^{\pi} - (1 - \quad) \bar{R}) \quad =$$

$$= (1 - \quad) \left[(1 - \quad ^{w}) w + \quad (w \quad ^{w\,''w} + \quad ^{\pi\,''w} - (1 - \quad) \bar{R}\,^{''w}) \right]$$

$$-$$

(68), (69), P 11.

P 12

(1 −), (1 −)= ↑,

, $^{\pi}$= ^{w} ↓.□